Invalid Ward-Takahashi identities and broken unitarity of the SM

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Abstract

It is found that in the SM the Ward-Takahashi(WT) identities of the axial-vector currents and the charged vector currents of fermions are invalid after spontaneous symmetry breaking. The spin-0 components of Z and W fields are revealed from the invalidity of these WT identities. The masses of these spin-0 components are at 10^{14}GeV. They are ghosts. Therefore, unitarity of the SM after spontaneous symmetry breaking is broken at 10^{14}GeV.

1 Introduction

The SM[1] is successful in many aspects. It is widely believed that the unitarity and renormalizability of the SM have been proved. A brief review of the history of the proof of the untarity and renormalizablity of the SM is presented.

1. Because of U(1) local gauge invariance and WT identities QED is unitary and renormalizable. In QED there is vector coupling between fermion and the gauge field.

2. Comparing with QED, the gauge fields in QCD are nonabelian. Using nonabelian gauge invariance and WT identities, in 1971 't Hooft[2] has proved that Yang-Mills field theory is unitary and renormalizable. In QCD between quarks and Yang-Mills fields there are vector couplings. Because of gauge invariance the WT identities including fermions can be naturally derived. Therefore, QCD is unitary and renormalizable.

3. In 1969 B.W. Lee and J.L.Gervais[3] have proved the linear σ model with a symmetry breaking term $c\sigma$ is unitary and renormalizable. It shows that the WT identities is not affected by the symmetry breaking term.

4. In 1971 't Hooft[4] studied a large set of different models in which local gauge invariance(including $SU(2)$) is broken spontaneously. The fields in all these models studied are massive, charged or neutral, spin one bosons, photons, and massive scalar particles.

Feynman rules and WT identities are found. The author concludes that "A renormalizable and unitary theory results, with photons, charged massive vector particles, and additional scalar particles." At the end of the paper fermions are introduced as

$$\mathcal{L}_N = -\bar{N}(m + \gamma_\mu D_\mu)N.$$

The fermions have the same mass and there is vector current only.

5. In 1972 B.W.Lee[5] has proved that Abelian gauge field with spontaneous symmetry breaking is renormalizable and unitary.

6. In the four papers By B.W Lee and Jean Zinn-Justin[6] renormalizability and unitarity of the theory of Yang-Mills fields with spontaneous symmetry breaking and scalar fields have been proved in detail. It shows that "spontaneous symmetry breaking doesn't affect the Ward-Takahashi identities". Therefore, the theory is renormalizable and unitary. No fermions are included in the studies of these papers. In their fourth paper[6](paper IV) the authors addresses the fermion problem "Recently Bardeen[7] has given a discussion of renormalizability of gauge theories with fermions. He has shown that the problem associated with anomalies in fermion loops can be isolated from the general problem of renormalizability and gauge invariance, and if fermion loop anomalies are absent, or canceled among them selves in lowest order, the presence of fermions does not hinder the WT identities from being valid."

The SM consists of gauga bosons, scalars, and fermions. The structure related to fermions is far more complicated than both QED and QCD. Between fermions and gauge bosons there are both vector and axial-vector couplings. Among vector couplings besides the neutral vector couplings there are charged vector couplings between fermions and W bosons. In both QED and QCD there are neither axial-vector couplings nor charged vector couplings. These new currents have not been studied in 't Hooft's[4] and Lee and Zinn-Justin's papers[6]. Anomaly affects WT identities. The anomalies are canceled out at the lowest order in the SM. According to Bardeen[7], in the SM anomaly doesn't affect the WT identities before and after spontaneous symmetry breaking.

In Ref.[8] we have presented a study on the effect of the vacuum polarization of Z and W bosons by fermions. The study[8] shows that after taking the vacuum polarization by massive fermions into account, both Z and W fields have a spin-0 components whose masses are about 10^{14}GeV. They are ghosts. Therefore, after spontaneous symmetry breaking unitarity of the SM is broken at the level of 10^{14}GeV. This result is in serious contradiction with the belief that the unitarity of the SM has been proved in Refs. [4,6]. In this paper we try to study the origin of the problem. The key point is that besides the anomaly are there other effects of the two new kinds of fermion currents, mentioned above, on the WT identities?

The paper is organized as 1)introduction; 2) WT identities of fermions; 3) vacuum polarization before spontaneous symmetry breaking; 4) vacuum polarization after spontaneous

symmetry breaking; 5) conclusions.

2 Ward-Takahashi identities of fermions

Without losing generality, we start the study from t- and b- quark generation. The Lagrangian of the SM for t and b quark generation is

$$\mathcal{L} = -\frac{1}{4}A^i_{\mu\nu}A^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{q}_L\{i\gamma\cdot\partial + \frac{g}{2}\tau_i\gamma\cdot A^i + g'\frac{Y}{2}\gamma\cdot B\}q_L$$
$$+\bar{q}_R\{i\gamma\cdot\partial + g'\frac{Y}{2}\gamma\cdot Bi\}q_R$$
$$+f_t\{\bar{q}_L t_R\phi_c + \phi^\dagger_c\bar{t}_R q_L\} + f_b\{\bar{q}_L b_R\phi + \phi^\dagger\bar{b}_R q_L\}, \tag{1}$$

where

$$q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}.$$

The couplings between fermions, Z, and W are

$$\mathcal{L} = \frac{\bar{g}}{4}\{(1-\frac{8}{3}\alpha)\bar{t}\gamma_\mu t + \bar{t}\gamma_\mu\gamma_5 t\}Z^\mu - \frac{\bar{g}}{4}\{(1-\frac{4}{3}\alpha)\bar{b}\gamma_\mu b + \bar{b}\gamma_\mu\gamma_5 b\}Z^\mu, \tag{2}$$

where $\alpha = sin^2\theta_W$,

$$\mathcal{L} = \frac{g}{2\sqrt{2}}\bar{t}\gamma_\mu(1+\gamma_5)bW^{+\mu} + +\bar{b}\gamma_\mu(1+\gamma_5)tW^{-\mu}. \tag{3}$$

Comparing with QED and QCD, the axial-vector currents and the charged vector currents are new.

Before spontaneous symmetry breaking the Lagrangian(1) is invariant under $SU(2)_L \times U(1)$ gauge transformation. From Eq.(1) following equations of fermion currents are found

$$\partial_\mu(\bar{t}\gamma_\mu t) = \frac{i}{2\sqrt{2}}g\{\bar{t}\gamma_\mu(1+\gamma_5)bW^-_\mu - \bar{b}\gamma_\mu(1+\gamma_5)tW^+_\mu\}$$
$$\partial_\mu(\bar{b}\gamma_\mu b) = \frac{i}{2\sqrt{2}}g\{\bar{b}\gamma_\mu(1+\gamma_5)tW^+_\mu - \bar{t}\gamma_\mu(1+\gamma_5)bW^-_\mu\},$$
$$\partial_\mu(\bar{t}\gamma_\mu t + \bar{b}\gamma_\mu b) = 0. \tag{4}$$

The axial-vector currents satisfy following equations in unitary gauge

$$\partial_\mu(\bar{t}\gamma_\mu\gamma_5 t) = \frac{i}{2\sqrt{2}}g\{\bar{t}\gamma_\mu\gamma_5 bW^-_\mu - \bar{b}\gamma_\mu\gamma_5 tW^+_\mu\} + i\sqrt{2}f_t\bar{t}\gamma_5 t\phi^0 + anomaly,$$
$$\partial_\mu(\bar{(b)}\gamma_\mu\gamma_5 b) = \frac{i}{2\sqrt{2}}g\{\bar{b}\gamma_\mu\gamma_5 tW^-_\mu - \bar{t}\gamma_\mu\gamma_5 bW^+_\mu\} + i\sqrt{2}f_b\bar{b}\gamma_5 b\phi^0 + anomaly. \tag{5}$$

It is well known when adding up all quarks and leptons, the anomalies are canceled out in the SM. Because of this cancellation we will not mention anomalies again in this paper. The equations of charged vector and axial-vector currents are derived

$$\partial_\mu(\bar{t}\gamma_\mu(1+\gamma_5)b) = -ie\bar{t}\gamma\cdot Ab - i\bar{g}(1-\alpha)\bar{t}\gamma_\mu(1+\gamma_5)bZ_\mu$$
$$+\frac{i}{\sqrt{2}}g\{\bar{t}\gamma_\mu(1+\gamma_5)t - \bar{b}\gamma_\mu(1+\gamma_5)b\}W^+_\mu - \frac{i}{\sqrt{2}}(f_b - f_t)\bar{t}b\phi^0 + \frac{i}{\sqrt{2}}(f_b+f_t)\bar{t}\gamma_5 b\phi^0. \tag{6}$$

Eqs.(4-6) are the WT identities in the form of operators. We apply these equations to study the vacuum polarizations of Z and W fields at the order of $O(\bar{g}^2)$ and $O(g^2)$ respectively.

In Refs.[4,6] the unitarity of the theory of nonabelian gauge fields and scalar fields with spontaneous symmetry breaking has been approved. In the SM fermion fields are included

and there are two new kinds of currents: axial-vector and charged vector currents. According to Refs.[4.6] the vacuum polarizations of Z and W fields by gauge bosons and Higgs don't have any effects on unitarity of the theory. Therefore, we only study the vacuum polarizations by fermions in this paper. In the vacuum polarizations the two new currents are involved.

3 Vacuum polarization before spontaneous symmetry breaking

Before spontaneous symmetry breaking at the lowest order the equations(4-6) become

$$\partial_\mu \bar{t}\gamma_\mu t = 0, \quad \partial_\mu \bar{t}\gamma_\mu\gamma_5 t = 0, \quad \partial_\mu \bar{b}\gamma_\mu b = 0, \quad \partial_\mu \bar{b}\gamma_\mu\gamma_5 b = 0,$$
$$\partial_\mu(\bar{t}\gamma_\mu(1+\gamma_5)b) = 0. \tag{7}$$

Before spontaneous symmetry breaking, the fermions of the SM are massless. Using the vertices(2,3), the amplitudes of the vacuum polarization of Z and W fields at one loop of fermions are calculated

$$\Pi^Z_{\mu\nu} = \frac{\bar{g}^2}{8}\frac{N_C}{(4\pi)^2}D\Gamma(2-\frac{D}{2})\int_0^1 dxx(1-x)(\frac{\mu^2}{L_0})^{\frac{\epsilon}{2}}(q_\mu q_\nu - q^2 g_{\mu\nu})(y_t+y_b),$$
$$\Pi^W_{\mu\nu} = \frac{g^2}{4}\frac{N_C}{(4\pi)^2}D\Gamma(2-\frac{D}{2})\int_0^1 dxx(1-x)(\frac{\mu^2}{L_0})^{\frac{\epsilon}{2}}(q_\mu q_\nu - q^2 g_{\mu\nu}), \tag{8}$$

where $y_t = 1+(1-\frac{8}{3}\alpha)^2$, $y_b = 1+(1-\frac{4}{3})^2$, $L_0 = m^2 - x(1-x)q^2$. m is a parameter introduced for the infrared divergence caused by massless quark. The amplitudes(8) show that the WT

identities(7) are satisfied before spontaneous symmetry breaking. The contributions of other generations of fermions can be obtained too. The WT identities are satisfied too.

4 Vacuum polarization after spontaneous symmetry breaking

After spontaneous symmetry breaking in unitary gauge there are

$$\phi^0 = \eta + H, \quad m_t = f_t\eta, \quad m_b = f_b\eta. \tag{9}$$

Substituting Eqs.(9) into Eqs.(4-6), new equations of fermion currents are obtained. After spontaneous symmetry breaking fermion mass appears in the equations of the axial-vector and charged vector currents. The fermion mass originates in the violation of $SU(2)_L \times U(1)$ symmetry.

The invalidity of the WT identities of fermion axial-vector and charged vector currents after spontaneous symmetry breaking can be shown clearly by the equations at the lowest order

$$\begin{aligned}
&\partial_\mu(\bar{t}\gamma_\mu\gamma_5 t) = 2im_t\bar{t}\gamma_5 t,\\
&\partial_\mu(\bar{b}\gamma_\mu\gamma_5 b) = 2im_b\bar{b}\gamma_5 b,\\
&\partial_\mu(\bar{t}\gamma_\mu(1+\gamma_5)b) = i(m_t - m_b)\bar{b}t + i(m_t + m_b)\bar{t}\gamma_5 b.
\end{aligned} \tag{10}$$

In the models studied in Refs.[4,6] there are no axial-vector and charged vector currents of fermions. Therefore, there are no such violations of WT identities(10).

We need to study the physical effects caused by the invalid WT identities(10). In this paper the vacuum polarization of Z and W fields at one loop of fermions are studied.

The amplitude of the vacuum polarization of Z fields by massive fermions at the lowest order is expressed as[8]

$$\Pi^Z_{\mu\nu} = \frac{1}{2}F_{Z1}(z)(p_\mu p_\nu - p^2 g_{\mu\nu}) + F_{Z2}(z)p_\mu p_\nu + \frac{1}{2}\Delta m^2_Z g_{\mu\nu}, \tag{11}$$

$$\begin{aligned}
F_{Z1} &= 1 + \frac{\bar{g}^2}{64\pi^2}\{\frac{D}{12}\Gamma(2-\frac{D}{2})[N_C y_q \sum_q (\frac{\mu^2}{m_q^2})^{\frac{\epsilon}{2}} + y_l \sum_l (\frac{\mu^2}{m_l^2})^{\frac{\epsilon}{2}}] \\
&\quad -2[N_C y_q \sum_q f_1(z_q) + y_l \sum_l f_1(z_l)] + 2[\sum_q f_2(z_q) + \sum_{l=e,\mu,\tau} f_2(z_l)]\}, \\
F_{Z2} &= -\frac{\bar{g}^2}{64\pi^2}\{N_C \sum_q f_2(z_q) + \sum_{l=e,\mu,\tau} f_2(z_l)\}, \\
\Delta m^2_Z &= \frac{1}{8}\frac{\bar{g}^2}{(4\pi)^2} D\Gamma(2-\frac{D}{2})\{N_c \sum_q m_q^2 (\frac{\mu^2}{m_q^2})^{\frac{\epsilon}{2}} + \sum_l m_l^2 (\frac{\mu^2}{m_l^2})^{\frac{\epsilon}{2}}\}.
\end{aligned} \tag{12}$$

where $y_q = 1 + (1-\frac{8}{3}\alpha)^2$ for $q = t, c, u$, $y_q = 1 + (1-\frac{4}{3}\alpha)^2$ for $q = b, s, d$, $y_l = 1 + (1-4\alpha)^2$, for $l = \tau, \mu, e$, $y_l = 2$ for $l = \nu_e, \nu_\mu, \tau_\mu$, $z_i = \frac{p^2}{m_i^2}$,

$$\begin{aligned}
f_1(z) &= \int_0^1 dx x(1-x) log\{1 - x(1-x)z\}, \\
f_2(z) &= \frac{1}{z}\int_0^1 dx log\{1 - x(1-x)z\}.
\end{aligned} \tag{13}$$

Comparing with Eq.(8), the expression of the vacuum polarization by massive fermions has two new terms: $F_{Z2}(z)$ and Δm^2_Z. Both the vector and axial-vector couplings contribute

to F_{Z1}. Only the axial-vector currents of massive fermion(2) contribute to F_{Z2} and Δm_Z^2. Eq.(12) show that in the limits, $m_{q(l)} \to 0$, we have

$$\Delta m_Z^2 = 0, \quad F_{Z2}(z) = 0.$$

Therefore, the two new terms are resulted in the violation of the WT identity of the axial-vector currents of fermions of the SM after spontaneous symmetry breaking. Because of these two new terms there is no current conservation in Eq.(11). The function F_{Z1} is used to renormalize the Z-field. In the SM the Z boson gains mass from the spontaneous symmetry breaking and Δm_Z^2 can be refereed to the renormalization of m_Z^2. F_{Z2} is finite and it indicates that Z field has nonzero divergence. $\partial_\mu Z^\mu$ can be defined as a spin-0 field ϕ_Z whose mass is labeled as m_{ϕ_Z} which can be determined. Therefore, the Z-field has the fourth independent spin-0 component ϕ_Z. We rewrite F_{Z2} as

$$F_{Z2}(z) = \xi_Z + (p^2 - m_{\phi_Z}^2) G_{Z2}(p^2), \tag{14}$$

G_{Z2} is the radiative correction, and

$$\xi_Z = F_{Z2}|_{p^2 = m_{\phi_Z}^2}. \tag{15}$$

The new free Lagrangian of the Z-field is constructed as

$$\mathcal{L}_{Z0} = -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \xi_Z (\partial_\mu Z^\mu)^2 + \frac{1}{2} m_Z^2 Z_\mu^2. \tag{16}$$

The term $(\partial_\mu Z^\mu)^2$ is dynamically generated and the coefficient ξ_Z is no longer a parameter of gauge condition and it can be determined (see the result below). It is necessary to emphasize that at $O(\bar{g}^2)$ the term $\xi_Z(\partial_\mu Z^\mu)^2$ of Eq.(16) is gauge independent. The new spin-0 field is defined as[8]

$$\phi_Z = \mp\frac{m_Z}{m^2_{\phi_Z}}\partial_\mu Z^\mu. \tag{17}$$

The equation of ϕ_Z is derived from Eq.(16) as

$$\partial^2\phi_Z - \frac{m^2_Z}{2\xi_Z}\phi_Z = 0. \tag{18}$$

Therefore, the mass of ϕ_z is defined as

$$m^2_{\phi_Z} = -\frac{m^2_Z}{2\xi_Z}. \tag{19}$$

Using the Eq.(15), $m^2_{\phi_Z}$ satisfies

$$2m^2_{\phi_Z}F_{Z2}|_{p^2=m^2_{\phi_Z}} + m^2_Z = 0. \tag{20}$$

From the expression(12), the value of $m^2_{\phi_Z}$ is determined by

$$3\sum_q \frac{m_q^2}{m_Z^2}z_q f_2(z_q) + \sum_l \frac{m_l^2}{m_Z^2}z_l f_2(z_l) = \frac{32\pi^2}{\bar{g}^2}. \tag{21}$$

For $z > 4$

$$f_2(z) = -\frac{2}{z} - \frac{1}{z}(1-\frac{4}{z})^{\frac{1}{2}}log\frac{1-(1-\frac{4}{z})^{\frac{1}{2}}}{1+(1-\frac{4}{z})^{\frac{1}{2}}} \tag{22}$$

is a very good approximation. Because of the ratios of $\frac{m_q^2}{m_Z^2}$ and $\frac{m_l^2}{m_Z^2}$ top quark dominates and the contributions of other fermions can be ignored. The equation has a solution at very large value of z. For very large z we have

$$\frac{2(4\pi)^2}{\bar{g}^2} + \frac{6m_t^2}{m_Z^2} = 3\frac{m_t^2}{m_Z^2} log\frac{m_{\phi_Z}^2}{m_t^2}. \tag{23}$$

The mass of the ϕ_Z is determined to be

$$m_{\phi_Z} = m_t e^{\frac{m_z^2}{m_t^2}\frac{16\pi^2}{3\bar{g}^2}+1} = m_t e^{28.4} = 3.78 \times 10^{14} GeV, \tag{24}$$

and

$$\xi_Z = -1.18 \times 10^{-25}.$$

The neutral spin-0 boson is extremely heavy. The Z-filed is decomposed as

$$Z_\mu = Z'_\mu \pm \frac{1}{m_Z}\partial_\mu \phi_Z, \tag{25}$$

$$\partial_\mu Z'^\mu = 0. \tag{26}$$

Using the Lagrangian(16), the propagator of Z boson is found

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2}\{-g_{\mu\nu} + (1 + \frac{1}{2\xi_Z})\frac{p_\mu p_\nu}{p^2 - m_{\phi_Z}^2}\}, \tag{27}$$

It can be separated into two parts

$$\Delta_{\mu\nu} = \frac{1}{p^2 - m_Z^2}\{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_Z^2}\} - \frac{1}{m_Z^2}\frac{p_\mu p_\nu}{p^2 - m_{\phi_Z}^2}. \tag{28}$$

The first part is the propagator of the physical spin-1 Z boson and the second part is the propagator of a new neutral spin-0 meson, ϕ_Z.

The minus sign of Eq.(28) indicates that the Fock space has indefinite metric and there is problem of negative probability when ϕ_Z is on mass shell. The ϕ_Z is a ghost field. Therefore, unitarity of the SM is broken at $E = m_{\phi_Z}$. This ghost field is dynamically generated by fermion loops and there is no way it can be canceled.

Similarly, the expression of the vacuum polarization of W-fields at one loop of fermions is calculated[8]

$$\Pi^W_{\mu\nu} = F_{W1}(p^2)(p_\mu p_\nu - p^2 g_{\mu\nu}) + 2F_{W2}(p^2)p_\mu p_\nu + \Delta m_W^2 g_{\mu\nu}, \tag{29}$$

where

$$\begin{aligned} F_{W1}(p^2) &= 1 + \frac{g^2}{32\pi^2} D\Gamma(2-\frac{D}{2})\int_0^1 dx x(1-x)\{N_C\sum_{iq}(\frac{\mu^2}{L_q^i})^{\frac{\epsilon}{2}} + \sum_{il}(\frac{\mu^2}{L_l^i})^{\frac{\epsilon}{2}}\} \\ &- \frac{g^2}{16\pi^2}\{N_C\sum_{iq} f^i_{1q} + \sum_{il} f^i_{1l}\} + \frac{g^2}{16\pi^2}\{N_C\sum_{iq} f^i_{2q} + \sum_{il} f_{2l}\}, \end{aligned} \tag{30}$$

$$F_{W2}(p^2) = -\frac{g^2}{32\pi^2}\{N_C\sum_{iq} f^i_{2q} + \sum_{il} f^i_{2l}\}, \tag{31}$$

$$\Delta m_W^2 = \frac{g^2}{4}\frac{1}{(4\pi)^2} D\Gamma(2-\frac{D}{2})\int_0^1 dx\{N_c\sum_{iq} L^i_q(\frac{\mu^2}{L^i_q})^{\frac{\epsilon}{2}} + \sum_{il} L^i_l(\frac{\mu^2}{L^i_l})^{\frac{\epsilon}{2}}\}. \tag{32}$$

where

$$L_q^1 = m_b^2 x + m_t^2(1-x), \quad L_q^2 = m_s^2 x + m_c^2(1-x), \quad L_q^3 = m_d^2 x + m_u^2(1-x), \tag{33}$$

$$L_l^1 = m_e^2 x, \quad L_l^2 = m_\mu^2 x, \quad L_l^3 = m_\tau^2 x,$$

$$f_{1q}^i = \int_0^1 dx x(1-x) log[1 - x(1-x)\frac{p^2}{L_q^i}] \tag{34}$$

$$f_{1l}^i = \int_0^1 dx x(1-x) log[1 - x(1-x)\frac{p^2}{L_l^i}], \tag{35}$$

$$f_{2q}^i = \frac{1}{p^2}\int_0^1 dx L_q^i log[1 - x(1-x)\frac{p^2}{L_q^i}], \tag{36}$$

$$f_{2l}^i = \frac{1}{p^2}\int_0^1 dx L_l^i log[1 - x(1-x)\frac{p^2}{L_l^i}]. \tag{37}$$

Unlike Eq.(8), the amplitude(29) doesn't satisfy current conservation after spontaneous symmetry breaking. There are two new terms in the amplitude(29), F_{W2} and Δm_W^2, which cause the violation of current conservation. These two new terms violate the WT identities(7). As shown in Eq.(10), the invalidity of the WT identity is caused by the charged currents of massive fermions. In fact, in the limit, $m_q(m_l) \to 0$, we obtain

$$F_{W2} = 0, \quad \Delta m_W^2 = 0.$$

The amplitude(29) goes back to the expression(8), current conservation is satisfied. Therefore, F_{W2} and Δm_W^2 are the results of the invalid WT identity(10). The function $F_{W1}(p^2)$ is used to renormalize the W-field. Δm_W^2 can be refereed to the renormalization of m_W^2. F_{W2} is finite and it makes nonzero divergence of W-fields. W fields have four independent components. According to Ref.[8], nonzero $\partial_\mu W^{\pm\mu}$ lead to the existence of two charged spin-0

states, $\phi_W^\pm$, whose mass is labeled as m_{ϕ_W} which can be determined. F_{W2} is rewritten as[8]

$$F_{W2} = \xi_W + (p^2 - m_{\phi_W}^2) G_{W2}(p^2), \tag{38}$$

$$\xi_W = F_{W2}(p^2)|_{p^2=m_{\phi_W}^2}, \tag{39}$$

where G_{W2} is the radiative correction of the term $(\partial_\mu W^\mu)^2$.

The free part of the Lagrangian of W-fields is redefined as

$$\mathcal{L}_{W0} = -\frac{1}{2}(\partial_\mu W^+\nu - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-) + 2\xi_W \partial_\mu W^{+^\mu} \partial_\nu W^{-^\nu}$$

$$+ m_W^2 W_\mu^+ W^{-\mu}. \tag{40}$$

In Ref.[8] the new $\phi_W^\pm$ fields are found to be

$$\phi_W^\pm = \mp \frac{m_W}{m_{\phi_W}^2} \partial_\mu W^{\pm\mu}. \tag{41}$$

They satisfy the equation

$$\partial^2 \phi_W^\pm - \frac{m_W^2}{2\xi_W} \phi_W^\pm = 0. \tag{42}$$

Therefore, the mass of $\phi_W^\pm$ is defined as

$$m_{\phi_W^\pm}^2 = -\frac{m_W^2}{2\xi_W} \tag{43}$$

Eq.(39) becomes

$$2m_{\phi_W}^2 F_{W2}(p^2)|_{p^2=m_{\phi_W}^2} + m_W^2 = 0 \tag{44}$$

Top quark dominates the Eq.(44)

$$\frac{p^2}{m_W^2}F_{W2} = -\frac{3g^2}{32\pi^2}\frac{m_t^2}{m_W^2}\{-\frac{3}{4}+\frac{1}{2z}+[\frac{1}{2}-\frac{1}{z}+\frac{1}{2z^2}]log(z-1)\}. \tag{45}$$

Eq.(45) has a solution at very large z

$$m_{\phi_W} = m_t e^{\frac{16\pi^2}{3g^2}\frac{m_W^2}{m_t^2}} = m_t e^{27} = 9.31\times 10^{13}GeV \tag{46}$$

and

$$\xi_W = -3.73\times 10^{-25}. \tag{47}$$

The propagator of W-field is derived from the Lagrangian(40)

$$\Delta^W_{\mu\nu} = \frac{1}{p^2-m_W^2}\{-g_{\mu\nu}+(1+\frac{1}{2\xi_W})\frac{p_\mu p_\nu}{p^2-m_{\phi_W}^2}\}, \tag{48}$$

and it can be separated into two parts

$$\Delta^W_{\mu\nu} = \frac{1}{p^2-m_W^2}\{-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_W^2}\} - \frac{1}{m_W^2}\frac{p_\mu p_\nu}{p^2-m_{\phi_W}^2}. \tag{49}$$

The first part is the propagator of physical spin-1 W-field and the second part is the propagator of the $\phi_W^\pm$ fields. The minus sign in front of the propagator of $\phi_W^\pm$ fields indicates that there is problem of indefinite metric and negative probability when $\phi_W^\pm$ are on mass shell. Therefore, $\phi_W^\pm$ are ghost fields. Unitarity of the SM is broken at $E\sim m_{\phi_W}$.

The study shows that the WT identities of axial-vector and charged vector currents are invalid after fermions gain masses from the spontaneous symmetry breaking. The invalidity

of these WT identities after spontaneous symmetry breaking generate three spin-0 ghosts whose masses are at 10^{14} Gev. These ghosts are dynamically generated by fermion loops and they cannot be canceled. Therefore, unitarity of the SM is broken at this energy level.

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